

Generating Functional and Large N-Limit of Nonlocal 2D Generalized Yang-Mills Theories ($nlgYM_2$'s)

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Abstract

Using the path integral method, we calculate the partition function and generating functional (of the field strengths) on the nonlocal generalized 2D Yang - Mills theories ($nlgYM_2$'s), which is nonlocal in auxiliary field [14]. Our calculations is done for general surfaces. We find a general expression for free energy of $W(\phi) = \phi^{2k}$ in $nlgYM_2$ theories at the strong coupling phase (SCP) regime ($A > A_c$) for large groups. In the specific ϕ^4 model, we show that the theory has a third order phase transition.

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1 Introduction

This paper will be devoted to a renewed study of two dimensional Yang-Mills theory without matter, a system which can be easily solved. Yet we will see that there is still much to say about this system. Pure two dimensional Yang-Mills theories(YM_2 's) have certain properties, such as invariance under area preserving diffeomorphism and lack of any propagating degrees of freedom[1]. There are, however, ways to generalize these theories without losing those properties. One way so - called generalized Yang-Mills theories(gYM_2 's) [2] is

$$iTr(B\epsilon^{\mu\nu}F_{\mu\nu}) + f(B) \tag{1}$$

Here $F_{\mu\nu}$ is the Yang-Mills field strength and B is a scalar field in the adjoint representation of the gauge group. Standard dimensional analysis applied to (1) gives $F_{\mu\nu}$ dimension 2 and B dimension 0, so power counting allows an arbitrary class function $f(B)$. This model produce by E. Witten [2] and has obtained the partition function by considering its action as a perturbation of the topological theory at zero area. In [3-5] the Green function, partition function and expectation values of Wilson loops were calculated. One can, however, use standard path - integration and calculate the observables of the theory [6,7]. To study the behaviour of these theories for large groups is also interest. This was studied in [8-11] for ordinary YM_2 theories and in [12,13] for gYM_2 theories. It was shown that YM_2 's and some classes of gYM_2 's have a third - order phase transition in a certain area. There is another way to generalize YM_2 and gYM_2 , and that is to use a non - local action for the auxiliary field, which so - called nonlocal $YM_2(nlYM_2$'s) and nonlocal $gYM_2(nlgYM_2$'s) theories, respectively [14]. The authors of [14] studied $nlYM_2$ and investigated the order of transition for that. We want study the wave function, partition function, generating functional of $nlgYM_2$ and also their properties for large gauge group in the state which $W(\phi) = \phi^4$. The scheme of the present paper is the following.

In sec.2, the wave function and partition function of $nlgYM_2$ on general surfaces are

computed. In sec.3, the generating functional of $nlgYM_2$ on disk and general surfaces are calculated. In sec.4, the properties of $nlgYM_2$ large groups, for the case which $f(B) = Tr(B^{2k})$, are studied. Finally in sec.5, we test our theory for ϕ^4 model ($f(B) = Tr(B^4)$). It is shown that the large group properties of $nlgYM_2$ are the same which was found for ordinary gYM_2 .

2 The Wave Function of $nlgYM_2$

The $nlgYM_2$ is defined by [14]

$$e^S := \int DB \exp \left\{ i \int Tr(BF) d\mu + \omega \left[\int f(B) d\mu \right] \right\}, \quad (2)$$

where $d\mu$ is the invariant measure of the surface

$$d\mu := \frac{1}{2} \epsilon_{\mu\nu} dx^\mu dx^\nu. \quad (3)$$

F is the field strength corresponding to the gauge field and B is a pseudo - scalar field in the adjoint representation of the group. Along the line of [7,14], we begin by calculating the wave function on a disk. we obtain

$$\psi_D(U) = \int DFe^S \delta \left(P \exp \oint_{\partial D} A, U \right). \quad (4)$$

Here U is the class of the Wilson loop corresponding to the boundary. The delta function is also a class delta function, in which, its support the boundary conditions. This delta function can be expanded in terms of the characters of irreducible unitary representations of the group; i.e.

$$\delta \left(P \exp \oint_{\partial D} A, U \right) = \sum_R \chi_R(U^{-1}) \chi_R \left(P \exp \oint_{\partial D} A \right). \quad (5)$$

We introduce Fermionic variables η and $\bar{\eta}$ in the representation R to write the Wilson loop as [6,7]

$$\chi_R \left(P \exp \oint_{\partial D} A \right) = \int D\eta D\bar{\eta} \exp \left\{ \int_0^1 dt \bar{\eta}(t) \dot{\eta}(t) + \oint_{\partial D} \bar{\eta} A \eta \right\} \eta^\alpha(0) \bar{\eta}_\alpha(1). \quad (6)$$

Inserting (6) in (5) and then (4), using the Schwinger - Fock gauge, and integrating over, F, B, and the Fermionic variables, respectively, one obtains

$$\psi_D(U) = \sum_R \chi_R(U^{-1}) d_R \exp\{\omega[AC_f(R)]\}, \quad (7)$$

Here d_R is the dimension of the representation R and

$$C_f(R)1_R =: f(-iT_R). \quad (8)$$

and $f(-iT_R)$ means that one has put $-iT^a$ in the representation R instead of B^a in the function f . Where as the action of the original B-F theory (2) is not extensive; i.e.

$$S_{A_1+A_2}(B, F) \neq S_{A_1}(B, F) + S_{A_2}(B, F). \quad (9)$$

Therefore, one cannot simply glue the disk wave function to obtain, the wave function corresponding to a larger disk. To obtain the wave function for an arbitrary surface, however, one can begin with a disk of the same area and impose boundary conditions on certain parts of the boundary of the disk. These conditions are those corresponding to the identifications needed for constructing the desired surface from a disk. The only things to be calculated are integrations over group of characters of the same representation [5]. This is easily done and one arrives at

$$\psi_{\sum_{g,q}}(U_1, \dots, U_n) = \sum_R h_R^q d_R^{2-2g-q-n} \chi_R(U_n^{-1}) \dots \chi_R(U_1^{-1}) \exp\{\omega[-C_f(R)A_{\sum_{g,q}}]\}, \quad (10)$$

where $\sum_{g,q}$ is a surface containing g handles, n boundaries and q projective planes. h_R is defined as

$$h_R := \int dU \chi_R(U^2), \quad (11)$$

$h_R = 0$ unless the representation R is self conjugate. In this case, this representation has an invariant bilinear form. Then, $h_R = 1$ if this form is symmetric and $h_R = -1$ if it is antisymmetric[15].

The partition function of the theory on a sphere is obtained if we put U_i 's equal to unity and g and q equal to zero. We obtain

$$Z_{s^2} = \sum_R d_R^2 \exp\{\omega[-AC_f(R)]\}. \quad (12)$$

3 The Generating Functional $Z[J]$ of $nlgYM_2$

To calculate the Green functions of the strength F^a 's, we again begin with the disk and calculate the wave function of $nlgYM_2$ on the disk, with a source term coupled to F ; i.e.

$$\psi_D[J] = \int DF e^{\{S + \int Tr(FJ)d\mu\}} \delta\left(Pexp \oint_{\partial D} A, U\right). \quad (13)$$

Following the same steps of the previous section, we arrive at

$$\psi_D[J] = \sum_R \chi_R(U^{-1}) Tr_R \left\{ Pexp \left(\omega \left[\int f(iJ^a(x) + iT^a) d\mu \right] \right) \right\}. \quad (14)$$

In the above equation P stands for ordering according to the angle variable on the disk. to obtain the generating functional $Z[J]$ of $nlgYM_2$ for an arbitrary surface, $\Sigma_{g,q}$, we can use the same procedure which was used in obtaining (10) and the result is

$$Z_{\Sigma_{g,q}}[J] = \sum_R h_R^q d_R^{2-2g-q-1} \exp\{\omega[AC_f(R)]\} Tr_R \left\{ Pexp \left(\omega \left[\int f(iJ^a + iT^a) d\mu \right] \right) \right\}. \quad (15)$$

As an example, consider YM_2 , in which $\omega[\int f(B)d\mu] = -\frac{1}{2}\epsilon \int Tr(B^2)d\mu$. In this case (15) reduces to

$$Z_{\Sigma_{g,q}}[J] = Z_1[J] \sum_R h_R^q d_R^{2-2g-q-1} \exp\left\{-\frac{\epsilon}{2} C_2(R) A_{\Sigma_{g,q}}\right\} Tr_R \left\{ Pexp \left(\epsilon \int dt \int ds \sqrt{g} J(t,s) \right) \right\}, \quad (16)$$

where

$$Z_1[J] = \exp \left(-\frac{\epsilon}{2} \int J^a J_a d\mu \right).$$

Which is in agreement with the result obtain in [7]. Functional differentiating of (15) with respect to $J(x)$ gives us the n -point functions of F 's in the Schwinger-Fock gauge.

4 Large N-Limit of $nlgYM_2$

Starting from (12), consider the case that gauge group is $U(N)$. The representation of this group are labeled by N integers n_i satisfying

$$n_i \geq n_j, \quad i \leq j. \quad (17)$$

The dimension of this representation is

$$d_R = \prod_{1 \leq i \leq j \leq N} \left(1 + \frac{n_i - n_j}{j - i}\right), \quad (18)$$

and the k -th Casimir is

$$C_k(R) = \sum_{i=1}^N [(n_i + N - i)^k - (N - i)^k]. \quad (19)$$

Taking $C_f(R)$ a linear function of the Casimirs (19) and redefine the function ω and introduce another function as

$$-N^2 V[A \sum_{k=1}^N a_k \hat{C}_k(R)] := \omega[-AC_f(R)], \quad (20)$$

where

$$\hat{C}_k(R) = \frac{1}{N^{k+1}} \sum_{i=1}^N (n_i + N - i)^k. \quad (21)$$

Then, following [10], we use the definitions

$$x := \frac{i}{N}, \quad (22)$$

and

$$\phi(x) = \frac{i - n_i - N}{N}. \quad (23)$$

So apart from an unimportant constant, the partition function takes the form

$$Z[\phi(x)] = \int D\phi(x) e^{\{-N^2 S(\phi)\}}, \quad (24)$$

where

$$S(\phi) = V\left(A \int_0^1 W[\phi(x)]dx\right) + \int_0^1 dx \int_0^1 dy \log|\phi(x) - \phi(y)|, \quad (25)$$

and

$$W(\phi) := \sum_{k=1} (-1)^k a_k \phi^k. \quad (26)$$

In the large N-limit, only the configuration of ϕ contributes to the partition function that minimizes S . To find it, we put variation of S with respect to ϕ equal to zero.

$$\frac{\hat{A}}{2} W'(\phi) = P \int_0^1 \frac{dt}{\phi(x) - \phi(x)}, \quad (27)$$

where

$$\hat{A} := AV' \left[A \int_0^1 dx W(\phi(x)) \right]. \quad (28)$$

One defines a density function for ϕ as

$$u(\phi) := \frac{dx(\phi)}{d\phi} \Big|_{\phi=z}, \quad (29)$$

which should be positive and normalized to

$$\int_{-a}^a u(z) dz = 1. \quad (30)$$

Then (27) becomes

$$\frac{\hat{A}}{2} W'(z) = P \int_{-a}^a \frac{u(t) dt}{z - t}. \quad (31)$$

To solve (31), we defined the function $H(z)$ on the complex z-plane [10]

$$H(z) := \int_{-a}^a \frac{u(t) dt}{z - t}. \quad (32)$$

This function is analytic on the complex plane, except for a cut at $[-a, a]$. With proceed the same procedure which was followed in [12], one arrives at

$$H(z) = \frac{\hat{A}}{2} W'(z) - \sqrt{z^2 - a^2} \sum_{m,n=0}^{\infty} M_n \frac{a^{2n} z^m}{(2n + m + 1)!} g^{(2n+m+1)}(0), \quad (33)$$

where

$$g(z) = \frac{\hat{A}}{2} W'(z), \quad (34)$$

and

$$M_n = \frac{(2n-1)!!}{2^n n!}, \quad M_0 = 1. \quad (35)$$

$g^{(k)}$ is the k -th derivative of g with respect to z . From (32), it is seen that

$$\text{Im}H(z + i\epsilon) = -\pi u(z), \quad x \in [-a, a] \quad (36)$$

which gives

$$u(z) = \frac{\sqrt{a^2 - z^2}}{\pi} \sum_{n,m=0}^{\infty} \frac{M_n a^{2n} z^m g^{(2n+m+1)}(0)}{(2n+m+1)!}. \quad (37)$$

To obtain a , one can use (30) and (37), which yields

$$\sum_{n=0}^{\infty} \frac{M_n a^{2n} g^{(2n-1)}(0)}{(2n-1)!} = 1. \quad (38)$$

Defining a free energy function as

$$F := -\frac{1}{N^2} S|_{\phi_{cla.}} \quad (39)$$

It is seen that

$$F'(A) = V'(A\kappa)\kappa, \quad (40)$$

where

$$\kappa = \int_0^1 W[\phi(x)]dx = \int_{-a}^a u(z)W(z)dz. \quad (41)$$

By making use of equations (37) and an explicit expression for $W(z)$ as a function of z , we can calculate κ and therefore at last we compute $F'_w(A)$ (40) for this model. Note that the above solution is valid in the weak ($A \leq A_c$) regime, where A_c is the critical area. If $A > A_c$, then the constraint $u \leq 1$ is violated.

5 The $W(z) = z^{2k}$ Model for $nlgYM_2$

5.1 WCP Regime($A \leq A_c$)

In order to study the behaviour of any model in the SCP regime ($A > A_c$), we need to know the explicit form of density function in the weak regime, $u_w(z)$. So by rewriting (37), (38) and (40) for z^{2k} model, one can arrives at

$$u_w(z) = \frac{k\hat{A}}{\pi} \sqrt{a^2 - z^2} \sum_{n=0}^{k-1} M_n a^{2n} z^{2k-2n-2}, \quad (42)$$

$$\frac{k\hat{A}a^{2k}}{2^k} Q(k) = 1, \quad (43)$$

$$F'_w(A) = \frac{kV'\hat{A}a^{4k}}{2^k} E(k), \quad (44)$$

where

$$Q(k) = \sum_{n=0}^{k-1} \frac{(2k-2n-3)!!(2n-1)!!}{(k-n-1)!(n+1)!}$$

$$E(k) = \sum_{n=0}^{k-1} \frac{(2k-2n-3)!!(2k+2n-1)!!}{(k-n-1)!(k+n+1)!} \quad (45)$$

This is, of course, in complete accordance with [13]. But one must now obtain the quantities in terms of A not \hat{A} . It is seen that

$$F'_w(A) = \frac{E(k)}{kAQ^2(k)} = \frac{1}{2kA}. \quad (46)$$

The function V is disappeared from $F'_w(A)$, as it can be seen by the rescaling $\hat{\phi} := A^{\frac{1}{2k}}\phi$. This completes our discussion of the weak-region $nlgYM_2$. As A increases, a situation is encountered where u_w exceeds 1. This density function is, however, not acceptable, as it violate the condition (17).

5.2 SCP Regime ($A > A_c$)

One of the interesting point of the $Z^{2k}(k > 1)$ model is the fact which the density function in weak-region (42) has only one minimum at $z = 0$, and two maxima which are symmetric with respect to origin [13]. So that to find the density function in strong-region,

will be relevant with three cut Cauchy problem. Hence following [12], we use the following ansatz for u_s

$$u_s(z) = \begin{cases} \hat{u}_s(z) & z \in L := [-a, -b] \cup [-c, c] \cup [b, a] \\ 1 & z \in L' := [-b, -c] \cup [c, b] \end{cases} \quad (47)$$

Using methods exactly the same as those used in [12], one must solve

$$\frac{\hat{A}}{2}W'(z) = P \int_{-a}^a \frac{u_s(t)dt}{z-t} \quad , \quad z \in L, \quad (48)$$

and

$$\int_c^b \left\{ \frac{\hat{A}}{2}W'(z) - P \int_{-a}^a \frac{u_s(t)dt}{z-t} \right\} dz = 0. \quad (49)$$

To do so, one defines a function H_s as

$$H_s(z) = \int_{-a}^a \frac{u_s(t)dt}{z-t}, \quad (50)$$

which is found to be

$$H_s(z) = k\hat{A}z^{2k-1} + 2T(z) \left[k\frac{\hat{A}}{2} \sum'_{[n_i]=0} \tau(n_1, n_2, n_3) z^{2n_4} - \int_c^b \frac{tdt}{(z^2 - t^2)T(t)} \right], \quad (51)$$

where the prime on the \sum indicate the following condition

$$\sum_{i=1}^4 n_i = k - 2, \quad (52)$$

and

$$T(z) = \sqrt{(a^2 - z^2)(b^2 - z^2)(c^2 - z^2)}, \quad (53)$$

$$\tau(n_1, n_2, n_3) = M_{n_1} M_{n_2} M_{n_3} a^{2n_1} b^{2n_2} c^{2n_3}. \quad (54)$$

Using the fact that $H_s(z)/T(z)$ should behave as $\frac{1}{z^4}$ for large z , one obtains

$$k\hat{A} \sum'_{[n_i]=0} \tau(n_1, n_2, n_3) = 2 \int_c^b \frac{tdt}{T(t)}, \quad (55)$$

$$k\hat{A} \sum'_{[n_i]=0} \tau(n_1, n_2, n_3) = 1 + 2 \int_c^b \frac{t^3 dt}{T(t)}. \quad (56)$$

Where the prime over summations in (55) and (56) indicates the following conditions, respectively

$$\sum_{i=1}^3 n_i = k - 1, \quad (57)$$

$$\sum_{i=1}^3 n_i = k. \quad (58)$$

In order to obtain the parameters a, b and c in spite of (55) and (56) we need another equation (50) which is found by expresses the action in terms of $u_s(z)$ and minimize that along with the (30), as a constraint [11,12]. By expanding (50) and (51) at large z and compare them, one can easily arrives at

$$F'_s(A) = V'_s(A\kappa_s) \left\{ k\hat{A} \sum'_{[n_i]=0} \tau(n_1, n_2, n_3) \tau_1(n_4, n_5, n_6) + 2 \sum'_{[n_i]=0} \tau_1(n_1, n_2, n_3) \int_c^b \frac{t^{2n_4+1} dt}{T(t)} \right\}, \quad (59)$$

where the prime over first and second summation indicate the following constraint, respectively

$$\sum_{i=1}^6 n_i = 2k, \quad (60)$$

$$\sum_{i=1}^4 n_i = k + 1, \quad (61)$$

and

$$\tau_1(n_1, n_2, n_3) = \frac{a^{2n_1} b^{2n_2} c^{2n_3}}{2^{n_1+n_2+n_3}} \prod_{i=1}^3 \frac{(2n_i - 3)!!}{n_i!}, \quad (62)$$

where, we define $(-3)!! = -1$.

Equation (59), is an explicit relation for $F'_s(A)$, which represents the SCP regime of our theory. It is seen that the structure of $F'_s(A)$ is very complicate, therefore, as example, we can study the order of transition for z^4 model(k=2).

6 The z^4 Model of $nlgYM_2$

6.1 WCP Regime ($A \leq A_c$)

In the previous section we study the $nlgYM_2$ for z^{2k} model. In this section we can check the result of it for z^4 model. By rewriting eqs.(42-45), we have

$$u_w(z) = \frac{\hat{A}}{\pi} \sqrt{a^2 - z^2} (a^2 + 2z^2), \quad (63)$$

$$\kappa_w = \frac{3a^4}{16} \quad (64)$$

$$\hat{A} = \frac{4}{3a^4} \quad (65)$$

and

$$F'_w(A) = \frac{1}{4A} \quad (66)$$

It is see that, the density function in WCP regime, $u_w(z)$, has a minimum at $z = 0$, and two maxima at $z_{1,2} = \pm \frac{a}{\sqrt{2}}$. Equations (63-66) are valid in the regime which $a \leq a_c = \frac{8}{3\sqrt{2}\pi}$ or $A \leq A_c$. The value of A_c , is obtained from

$$u_w(z_{1,2}) = 1, \quad (67)$$

which gives

$$A_c V'_c \left(\frac{32A_c}{27\pi^4} \right) = \frac{27\pi^4}{256}. \quad (68)$$

In spite of some constant, these almost are the same results which have been calculated for local gYM_2 theory [12].

6.2 SCP Regime ($A > A_c$)

The $z^4(z)$ model for $nlgYM_2$, is a state which, the density function in WCP have a minimum at origin and two maxima which are symmetric with respect to origin. So one can use of the results in previous section and arrives at

$$\int_c^b \left\{ 2\hat{A}z^3 - P \int_{-a}^a \frac{u_s(t)dt}{z-t} \right\} dz = 0, \quad (69)$$

$$\hat{A}(a^2 + b^2 + c^2) = 2 \int_c^b \frac{tdt}{T(t)}, \quad (70)$$

$$\hat{A} \left\{ (a^2b^2 + a^2c^2 + b^2c^2) + \frac{3}{2}(a^4 + b^4 + c^4) \right\} = 2 + 4 \int_c^b \frac{t^3 dt}{T(t)}. \quad (71)$$

Finally by making use of eqs.(54,59,62), it is seen that

$$\begin{aligned} F'_s(A) = V'(A\kappa_s) & \left\{ \frac{\hat{A}}{16} \left[\frac{5}{4}(a^8 + b^8 + c^8) - \frac{1}{2}(a^4b^4 + a^4c^4 + b^4c^4) + (a^2b^2c^4 + a^2c^2b^4 + a^4b^2c^2) \right. \right. \\ & \left. \left. - (a^2b^6 + a^2c^6 + b^2a^6 + b^2c^6 + c^2a^6 + c^2b^6) \right] \right. \\ & \left. + \frac{1}{8} \left[a^6 + b^6 + c^6 - (a^2b^4 + a^2c^4 + b^2a^4 + b^2c^4 + c^2a^4 + c^2b^4) + 2a^2b^2c^2 \right] \int_c^b \frac{t dt}{T(t)} + \right. \\ & \left. \frac{1}{4} \left[a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2) \right] \int_c^b \frac{t^3 dt}{T(t)} + (a^2 + b^2 + c^2) \int_c^b \frac{t^5 dt}{T(t)} - 2 \int_c^b \frac{t^7 dt}{T(t)} \right\} \quad (72) \end{aligned}$$

By using the same procedure was used in [12,13] and expand eqs.(69-72) near the critical point and then solve them with together, we can obtain

$$F'_s(A) = \frac{V'_s}{A} \left[\frac{1}{4} + \frac{\beta}{27} \alpha^2 + \dots \right], \quad (73)$$

or

$$F'_s(A) - F'_w(A) = \frac{\beta}{27A_c} \alpha^2 + \dots, \quad (74)$$

where

$$\alpha = \left(\frac{A - A_c}{A_c} \right)^2, \quad (75)$$

and

$$\beta = \left(1 + \frac{A_c \kappa_{cs} V''_{cs}}{V'_{cs}} \right)^2. \quad (76)$$

It is seen that the theory, for ϕ^4 model has a third order phase transition, which is in agreement with ordinary gYM_2 .

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